Problem 2 Pop-Pop Boat

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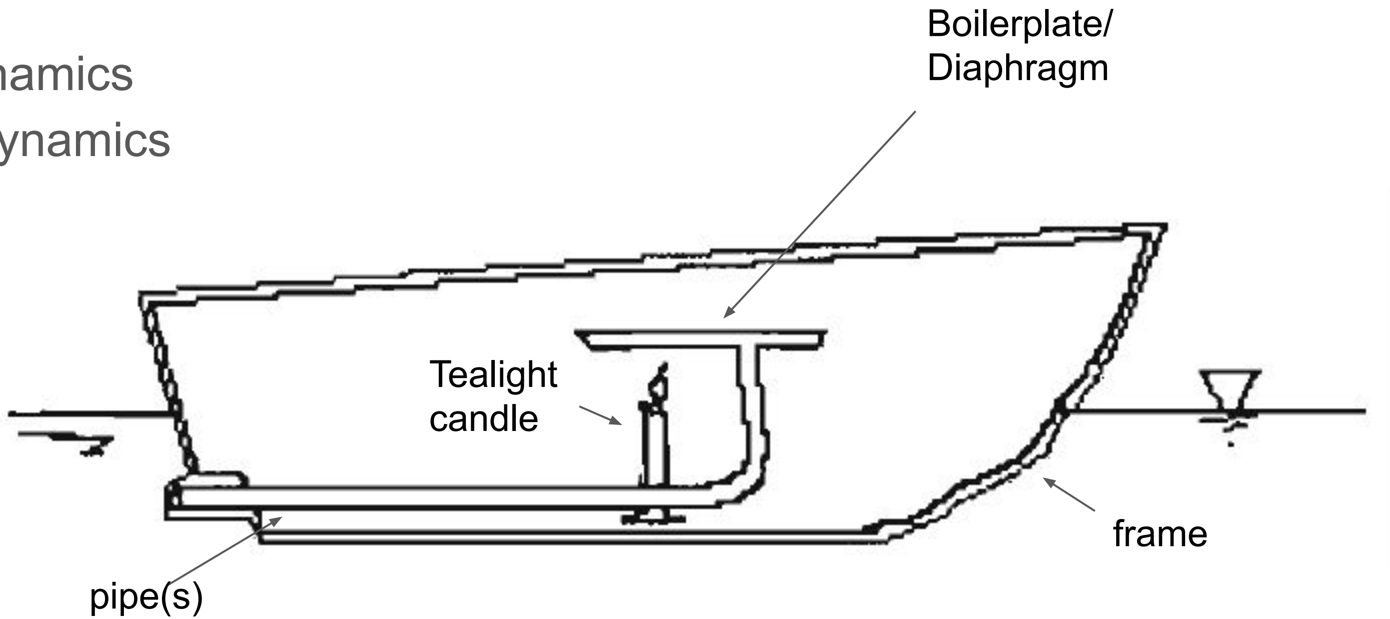
Prompt

The Pop-Pop boat is a small toy powered by a candle. Its engine is very simple, since it is just made of a boiler.

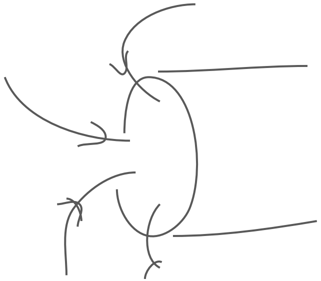
1. Propose a boat design that **maximizes the travelled distance** using a tealight candle.
2. Estimate the **energy conversion efficiency** of your boat. (engine: traveled)

Theory: What is a pop-pop boat?

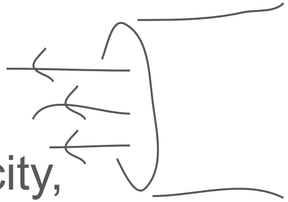
1. Hydrodynamics
2. Thermodynamics



Theory: Hydrodynamics



- 2 phases: suction, exhaust
  - Boat starts w/ water in pipes/boiler, so net mass flux = 0
  - But divergence of inflow greater than outflow ⇒ net thrust > 0
    - Due to viscosity & pressure



- By considering momentum conservation at the equilibrium velocity,

$$< thrust > = \rho \alpha < U_e^2 >$$

Density	: ρ
Area of the exit plane	: a
Jet velocity	: U_e

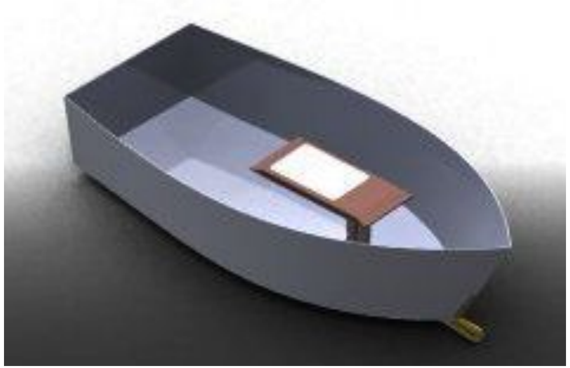
Theory: Thermodynamics

- Candle ⇒ steam ⇒ pressurizes boilerplate ⇒ forces water out of pipes
- Steam travels down pipes to cooler regions ⇒ condenses ⇒ pressure drops ⇒ draws water into pipes

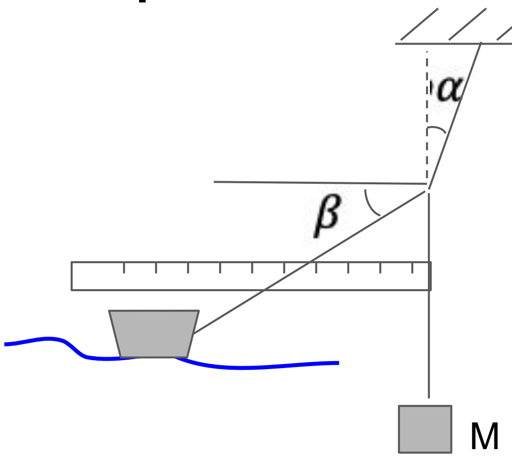
Key Parameters

- No effect
  - putt-putt noise does not affect velocity (e.g. thickening diaphragm)
  - Flexibility of diaphragm (flexible vs inflexible)
- Significant effect (thrust)
  - Number of pipes
  - Length of pipes
  - Jet velocity
  - Jet exit cross-sectional area
  - Reservoir temperature

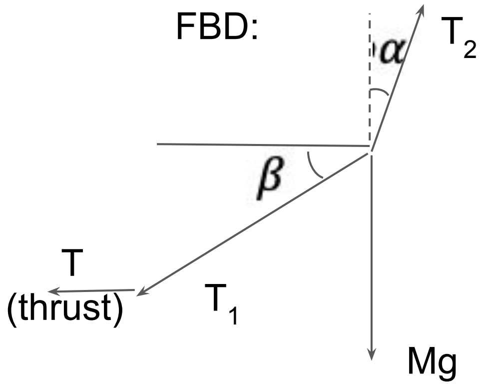
Experimental Setup: Boat Build



Experimental Setup: Thrust



FBD:



- Approximate: if α is small (<5 degrees) & $T_2 << T_1$ then $T \approx Mg\,\alpha$
- So run boat and stack weights until equilibrium achieved

Experimental Setups for specific factors

- Introduce bubble into clear pipe extensions w/ high-speeds recording
    - bubble displacement ⇒ suction & exhaust velocities, pressure at pipe exit

Data Analysis

1. Work done by boat: $< T > = \rho\alpha < U_e^2 >$
2. Work done by steam:
    a. Area under P-V diagram

Future

1. Maximizing distance traveled
    a. Build actual boat(s) and implement experimental setups
        i. Geometry of heating region (boilerplate vs coiled)
2. Energy efficiency
    a. Calculate efficiency for specific boat(s) we build
    b. Account for energy loss of tealight candle itself?

References

- https://youtu.be/PKk3w1M4J-4?t=519
- https://youtu.be/0ki9Kta8g14
- Propulsion of the Putt-Putt Boat - 1 by V Sharadha
- Propulsion of the Putt-Putt Boat - 2 by V Sharadha
- “2 cheap means to measure the thrust of a pop-pop engine” by Jean-Yves
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Hydrodynamic Calculations

Variable description:		
Density	:	ρ
Area of the exit plane	:	a
Acceleration of the reference frame	:	a_{rf}
Volume of the control volume	:	V
Surface forces	:	F_s
Body forces	:	F_b
Velocity of the mass leaving the control volume	:	U
Drag force	:	D
Velocity of the boat	:	U_∞
Jet velocity	:	U_e

Using the momentum conservation equation,

$$F_s + F_b - \int_V a_{rf}\, \rho\, dV = \int_{cs} U\, \rho U \cdot da + \frac{\partial}{\partial t}\left(\int_V U\, \rho\, dV\right).$$

After the boat has attained uniform velocity, along the x-direction,

$$F_b = 0$$
$$F_x = \int \tau\, da.i = D$$
$$a_{rf} = 0$$

Upon averaging over a cycle of operation,

$$\frac{\partial}{\partial t}\left(\int_V U\, \rho\, dV\right) = 0.$$

Using the notation,

$$\frac{1}{T}\int_0^T G\, dt = \langle G \rangle,$$

the momentum conservation equation becomes,

$$\langle F_{sx} \rangle = \int_{cs} \langle U.i\, \rho\, U.da \rangle + \int_{cs} \langle p\, da.i \rangle.$$

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During outflow, this equation becomes

$$\int_a \langle U_e.i\; \rho\, U_e.da\rangle + \int_a \langle p\; da.i\rangle\; \rho a\langle U_e\rangle^2.$$

During suction,

$$\int_a \langle U_i.i\; \rho\, U_e.da\rangle + \int_a \langle p\; da.i\rangle = 0.$$

Therefore the average drag force $\langle D\rangle \approx \rho a\langle U_e^2\rangle = \langle thrust\rangle$, under steady-state conditions. Drag can also be given in terms of the drag coefficient C_D, as,

$$D = C_D\,(\frac{1}{2}\rho U_\infty^2 A),$$ where C_D can be obtained from correlations.